Calculus (Type A)

Part I: Multiple-answer questions. For these questions, your answer should be one or more. To score all the points for each question, you must select ALL of the correct answers and NONE of the incorrect answers. Missing a correct answer or taking an incorrect one, you will lose 3 points. In the other cases, you will get zero.

1. (7pts.) Which of the following definitions are **correct**?

   Ⓐ $\lim_{n\to\infty}\sum_{i=1}^{n}\sqrt{1+\frac{i}{n}}\,\frac{2}{n}=\int_0^2\sqrt{1+x}\,dx$

   Ⓑ $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{i}{n}\sqrt{1+\frac{i}{n}}\,\frac{1}{n}=\int_0^1 x\sqrt{1+x}\,dx$

   Ⓒ $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{i}{n}\sqrt{1+\frac{i}{n}}\,\frac{1}{n}=\int_1^2(x-1)\sqrt{x}\,dx$

   Ⓓ $\lim_{n\to\infty}\sum_{i=1}^{n}\sqrt{1+\frac{2i}{n}}\,\frac{2}{n}=\int_1^3\sqrt{x}\,dx$

   Ans: B, C, D

2. (7pts.) Which of the following series are **correct**?

   Ⓐ The average value of $f(x)=\frac{1}{x^2}$ on $[1, 2]$ is $\frac{1}{2}$.

   Ⓑ The average value of $f(x)=x^2\sin x$ on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ is $\frac{\pi^2}{2}$

   Ⓒ If $f(x)$ is concave upward on $[a, b]$, then $f_{avg} > f(\frac{a+b}{2})$.

   Ⓓ If $f(x)$ is concave downward on $[a, b]$, then $f_{avg} < f(\frac{a+b}{2})$.

   Ans: A, C, D

3. (9pts.) Which of the following definite integrals are **correct** ?

   Ⓐ $\int_0^{\pi} x\sin x\,dx=\pi$

Ⓑ $\int_0^{\pi/2} x \sin x \, dx = 1$

Ⓒ $\int_0^{\pi} x^2 \sin x \, dx = \pi^2 - 4$

Ⓓ $\int_0^{\frac{\pi}{2}} \sin^{-1} x dx = \frac{\pi}{2}$

Ans: A, B, C,

4. (10pts.) Let V represent the volume of the solids obtained by rotating the region bounded by the curves $y = f(x),\ y = 0,\ x = a, x = b$ about the *y*-axis. Which of the following statements are **correct**?

Ⓐ If $f(x) = \sqrt[4]{x},\ a = 0$ and $b = 1$, then $V = \frac{8\pi}{9}$

Ⓑ If $f(x) = \sin x,\ a = 0$ and $b = \pi$, then $V = 2\pi^2$

Ⓒ If $f(x) = x,\ a = 0$ and $b = 1$, then $V = \frac{2\pi}{3}$

Ⓓ If $f(x) = 1/x,\ a = 0$ and $b = 1$, then $V = 2\pi$

Ans: A, B, C, D

5. (8pts.) Which of the following statements are **correct**?

Ⓐ $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$

Ⓑ If $f(x) = \tan^{-1}x$, then $f'(2) = \frac{1}{3}$.

Ⓒ If $f(x) = x^{\cos x}$ for $x > 0$, then $f'(x) = x^{\cos x} \cdot (-\sin x)$.

Ⓓ $\sin^{-1}\left(\cos\frac{1}{2}\right) = \frac{\pi - 1}{2}$

Ans: A, D

6. (8pts.) Let $S = \lim_{x \to 0^+} (1 + \sin 2x)^{1/x}$. Which of the following statement are **correct**?

Ⓐ $7 > S > 1$

Ⓑ $2 > S > 0$

Ⓒ $3 > S \geq 1$

Ⓓ $9 > S > 5$

Ans: D.

7. (7pts.) Which of the following statements are **incorrect**?

Ⓐ The series $\sum_{n=1}^{\infty} ne^{-n}$ diverges.

Ⓑ The series $\sum_{n=1}^{\infty} \frac{\tan^{-1} n}{1+n^2}$ diverges.

Ⓒ The series $\frac{1}{2}+\frac{1}{5}+\frac{1}{10}+\frac{1}{17}+\frac{1}{26}+\cdots$ converges.

Ⓓ The series $\sum_{n=2}^{\infty} \frac{\cos n}{n \ln n}$ diverges .

Ans: A, B

8. (7pts.) Find the Maclaurin series for $f(x) = x\cos x$. Which of the following series expansion is **correct**?

Ⓐ $\sum_{n=0}^{\infty} \frac{(-1)^{n+2}}{(2n)!} x^{2n+1}$

Ⓑ $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(2n-1)!} x^{2n-1}$

Ⓒ $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(2n-2)!} x^{2n-1}$

Ⓓ None

Ans: A,C

9. (7pts.) Which of the following is always **true**?

Ⓐ If $f(x) = \lim_{n\to\infty} S_n(x)$ where $S_n(x) = a_1(x) + a_2(x) + \cdots + a_n(x)$ and is differentiable for all $x \in \mathrm{R}$, then $f'(x) = \lim_{n\to\infty} S_n'(x)$.

Ⓑ If a power series $\sum_{n=0}^{\infty} a_n x^n$ has a radius of convergence $R = 3$, then it converges at x = 3.

Ⓒ There is no such a power series that converge only on the interval $[0, \infty)$.

Ⓓ None

Ans: C

Part II：Choices. Select only **ONE** answer choice from a list of four choices. (5% for each)

1. $\lim_{x\to 0^+} \frac{e^x-1-x}{x}$ = Ⓐ 0 Ⓑ 1 Ⓒ $\frac{1}{2}$ Ⓓ $\frac{1}{4}$

   Ans: A

2. $\lim_{x\to 0} \frac{4x^2\cos x}{\sin^2 3x}$ = Ⓐ 1 Ⓑ 0 Ⓒ $\frac{4}{9}$ Ⓓ $\frac{4}{3}$

   Ans: C

3. $\int_0^{\infty} e^{-x}\cos^2 x\,dx$ = Ⓐ 1 Ⓑ $\frac{1}{2}$ Ⓒ $\frac{3}{5}$ Ⓓ $\frac{2}{3}$

   Ans: C

4. $\int_2^{\infty} \frac{1}{x\cdot\ln^2 x}dx$ = Ⓐ $\frac{1}{2}$ Ⓑ $\ln 2$ Ⓒ $\frac{2}{e}$ Ⓓ none

   Ans: D

5. $\sum_{n=0}^{\infty} \frac{1}{(n+1)!}$ = Ⓐ *e-1* Ⓑ 2 Ⓒ *e* Ⓓ $\frac{3}{2}$

   Ans: A

6. Let A represent the area enclosed by one loop of polar curve $r = \cos 3\theta$. Which of the following is **true**?

   Ⓐ A=$\frac{\pi}{12}$ Ⓑ A=$\frac{\pi}{8}$ Ⓒ A=$\frac{\pi}{6}$ Ⓓ A=$\frac{\pi}{4}$

   Ans: A